Fractional Calculus & Applied Analysis
An International Journal for Theory and Applications
VOLUME 6, NUMBER 4 (2003) ISSN 1311-0454

HARDY-LITTLEWOOD-STEIN-WEISS INEQUALITY IN THE LEBESGUE SPACES WITH VARIABLE EXPONENT

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Dedicated to Professor Paul Butzer,
on the occasion of his 75th birthday

Abstract

The generalization

$$\left\| I^{\alpha(\cdot)} f \right\|_{L^{q(\cdot)}(\Omega,|x-x_0|^{\mu})} \le C \, \|f\|_{L^{p(\cdot)}(\Omega,|x-x_0|^{\gamma})}$$

of the Hardy-Littlewood-Stein-Weiss inequality is proved for the potential operator $I^{\alpha(\cdot)}f(x) = \int\limits_{\Omega} \frac{f(y)\,dy}{|x-y|^{n-\alpha(x)}}$ in the spaces $L^{p(\cdot)}(\Omega)$ with variable exponent $p(x)$ in the case of bounded domains Ω in R^n, where $x_0 \in \overline{\Omega}$, $\frac{1}{q(x)} \equiv \frac{1}{p(x)} - \frac{\alpha(x)}{n}$, $\alpha(x_0)p(x_0) - n < \gamma < n[p(x_0) - 1]$ and $\mu = \frac{q(x_0)}{p(x_0)}\,\gamma$.

Mathematics Subject Classification: 46E35

Key Words and Phrases: Hardy-Littlewood inequality, Stein-Weiss inequality, Lebesgue spaces with variable exponents, Riesz potentials, maximal functions